The physiological differentiation we can trace when we see that the eosinophile cell has accentuated the glandular and protective character of the primitive cell; while in its attack by direct contact brought about by pseudopodial activity we see the remnant of the direct pseudopodial and ingestive attack of the primitive cell.

The hyaline cell, or permanently free phagocyte, represents the specialisation of the direct pseudopodial ingestive activity of the primitive cell.

While, lastly, the absorptive powers of the primitive cell are represented by the rose-staining cell of the more differentiated animal forms.

II. "Stability and Instability of Viscous Liquids." By A. B. Basset, M.A., F.R.S. Received October 10, 1892.

(Abstract.)

The principal object of this paper is to endeavour to obtain a theoretical explanation of the instability of viscous liquids, which was experimentally studied by Professor Osborne Reynolds.*

The experiment, which perhaps most strikingly illustrates this branch of hydrodynamics, consisted in causing water to flow from a cistern through a long circular tube, and by means of suitable appliances a fine stream of coloured liquid was made to flow down the centre of the tube along with the water. When the velocity was sufficiently small, the coloured stream showed no tendency to mix with the water; but when the velocity was increased, it was found that as soon as it had attained a certain critical value, the coloured stream broke off at a certain point of the tube and began to mix with the water, thus showing that the motion was unstable. It was also found that as the velocity was still further increased the point at which instability commenced gradually moved up the tube towards the end at which the water was flowing in.

Professor Reynolds concluded that the critical velocity W was determined by the equation

$$\mathrm{W}a\rho/\mu < n,$$

where a is the radius of the tube, ρ the density, and μ the viscosity of the liquid, and n a number; but the results of this paper show that this formula is incomplete, inasmuch as it does not take any account of the friction of the liquid against the sides of the tube.

In the first place, if the surface friction is supposed to be zero, so that perfect slipping takes place, the motion is stable for all veloci-

* 'Phil. Trans.,' 1883, p. 935.

ties. If ϵ^{kt} be the time factor of a disturbance of wave-length λ, the value of k is

$$k = -\frac{2\iota\pi W}{\lambda} - \frac{\mu}{\rho a^2}\left(\frac{4\pi^2 a^2}{\lambda^2} + n^2\right) \quad \ldots\ldots\ldots\ldots (1),$$

where n is a root of the equation $J_1(n) = 0$.

Experiment shows that when the velocity is greater than about 6 inches per second, the frictional tangential stress of water in contact with a fixed or moving solid is approximately proportional to the square of the relative velocity. This introduces a constant β, which may be called the coefficient of sliding friction, whose dimensions are $[ML^{-3}]$, and are therefore the same as those of a density. This constant may have any positive real value; $\beta = 0$ corresponding to perfect slipping or zero tangential stress, whilst $\beta = \infty$ corresponds to no slipping, which requires that the velocity of the liquid should be the same as that of the surface with which it is in contact. Owing to the intractable nature of the general equations of motion of a viscous liquid, I have been unable to obtain a complete solution, except on the hypothesis that β is an exceedingly small quantity. This supposition, I fear, does not represent very accurately the actual state of fluids in contact with solid bodies; but, at the same time, the solution clearly shows that the instability observed by Professor Reynolds does not depend upon viscosity alone, but is due to the action of the boundary upon a *viscous* liquid.

To a first approximation, the real part of k is proportional to

$$\frac{Wa\beta}{\mu} - \frac{(n^2+m^2a^2)^2}{4n^2} \quad \ldots\ldots\ldots\ldots\ldots\ldots (2),$$

where $2\pi/m$ is the wave-length of the disturbance, and n is a root of the equation $J_1(n) = 0$. Since the second term is a number, this shows that the motion will be stable, provided

$$Wa\beta/\mu < \text{a number}.$$

The experiments of Professor Reynolds conclusively show that the critical velocity at which instability commences is proportional to μ/a; and the fact that the theoretical condition of stability turns out to be that Wa/μ, multiplied by a quantity of the same dimensions as a density, should be less than a certain number, appears to be in substantial agreement with his experimental results.

The results of the investigation may be summed up as follows:—

(i.) *The tendency to instability increases as the velocity of the liquid, the radius of the tube, and the coefficient of sliding friction increase; but diminishes as the viscosity increases.*

(ii.) *The tendency to instability increases as the wave-length* ($2\pi/m$) *of the disturbance increases.*

The remainder of the paper is occupied with the discussion of a variety of problems relating to jets and wave motion.

I find that when a cylindrical jet is moving through the atmosphere, the tendency of the viscosity of the jet is always in the direction of stability. The velocity of the jet does not affect the stability unless the influence of the surrounding air is taken into account; if, however, this is done, it will be found that it gives rise to a term proportional to the product of the density of the air and the square of the velocity of the jet, whose tendency is to render the motion unstable. The tendency of surface tension (as has been previously shown by Lord Rayleigh) is in the direction of stability or instability according as the wave-length of the disturbance is less or greater than the circumference of the jet.

If, in addition, the jet is supposed to be electrified, the condition of stability contains a term proportional to the square of the charge multiplied by a certain number, n. When the ratio of the circumference of the jet to the wave-length is less than 0·6, n is positive, and the electrical term tends to produce stability; but when this ratio is greater than 0·6, n is negative, and the electrical term tends to produce instability. It must, however, be recollected that when the above ratio is greater than unity the tendency of surface tension is to produce stability; but if the influencing body is capable of inducing a sufficiently large charge, the electrical term (when $2\pi a > \lambda$) will neutralize the effect of surface tension and viscosity, and the motion will be unstable.

The well-known calming effect of "pouring oil on troubled waters" has passed into a proverb. The mathematical investigation of this phenomenon is as follows:—The oil spreads over the water so as to form a very thin film; we may therefore suppose that the thickness l of the oil is so small compared with the wave-length that powers of l higher than the first may be neglected. Also, since the viscosity of olive oil in C.G.S. units is about* 3·25, whilst that of water is about 0·014, the former may be treated as a highly viscous liquid, and the latter as a frictionless one.

The result is as follows:—

Let ρ_1, ρ be the densities of the water and oil, T_1 the surface tension between oil and water, T the surface tension between oil and air, μ the viscosity of the oil, and ϵ^{kt} the time factor, then, to a first approximation,

$$k = -\frac{\{g(\rho_1-\rho)+T_1 m^2\}(g\rho - T m^2)l}{4\mu\{g\rho_1-(T-T_1)m^2\}}.$$

For olive oil, $T_1 = 20{\cdot}56$, $T = 36{\cdot}9$, so that $T > T_1$; and I find that

* Osborne Reynolds, 'Phil. Trans.,' 1886, p. 171.

the motion will be stable unless the wave-length of the disturbance lies between about 9/11 and 6/5 of a centimetre. This result satisfactorily explains the effect of oil in calming stormy water.

III. "On the Colour of the Leaves of Plants and their Autumnal Changes." By ARTHUR HILL HASSALL, M.D. Lond. Communicated by the Rt. Hon. Professor HUXLEY, F.R.S. Received June 21, 1892.

IV. "Observations on the Earthquake Shocks which occurred in the British Isles and France during the month of August, 1892." By EDWARD HULL, F.R.S., F.G.S., Professor of Geology in the Royal College of Science. Received October 5, 1892.

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